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JHEP10(2009)082

(http://iopscience.iop.org/1126-6708/2009/10/082)

Published by IOP Publishing for SISSA

Received: *August 20, 2009*
Revised: *October 6, 2009*
Accepted: *October 8, 2009*
Published: *October 27, 2009*

JHEP10(2009)082

Spectral flow of the non-supersymmetric microstates of the D1-D5-KK system

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Abstract: We show that a realisation of spectral flow as a coordinate transformation for asymptotically four-dimensional solutions can be extended to the non-supersymmetric case. We apply this transformation to smooth geometries describing microstates of the D1-D5-KK monopole system in type IIB supergravity compactified on a six-torus, and obtain solutions with an additional momentum charge. We study the supersymmetric and near-core limits of this construction.

Keywords: Black Holes in String Theory, Black Holes

ArXiv ePrint: 0908.0417

doi:10.1088/1126-6708/2009/10/082

Contents

1 Introduction

Supersymmetric D-brane systems with a large degeneracy of ground states have been a central element in progress in string theory as a quantum theory of gravity. One of the theory's great successes is the discovery that counting such states at weak coupling reproduces the black hole entropy [1, 2]. The AdS/CFT correspondence discovered by considering a near-horizon limit of the brane systems [3] then led to a significantly better understanding of the relation between geometry and D-brane descriptions. Further understanding of this relation has come from the construction of smooth horizon-free geometries corresponding to individual states of the D-brane system, showing that the map between geometry and field theory can go beyond the thermodynamic regime [4–7]. Though a generic microstate is expected to admit a description only in the full string theory (as found recently in [8]), large classes of geometries dual to supersymmetric microstates of a three-charge brane system in five dimensions and a four-charge brane system in four dimensions have been constructed. Mathur and his collaborators, in a series of papers, have argued that the information paradox could be resolved if the black hole geometry is viewed as a coarse grained description, averaging over geometries describing the individual microstates which differ in a 'fuzz ball' region inside the would-be horizon of the black hole. See [9, 10] for reviews of this work.

To test this proposed description of black holes and to further improve our understanding of the relation between geometry and D-branes, it is useful to construct smooth geometries dual to the non-supersymmetric excited states of the D-brane systems. This allows us to consider dynamical issues involving transitions between different states. Very few such geometries have been constructed. Geometries dual to two- and three-charge brane systems which are asymptotically flat in five dimensions were constructed in [11], and geometries dual to three-charge brane systems which are asymptotically flat in four dimensions were constructed in [12, 13]. These geometries are smooth in the duality frame where the brane

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charges correspond to a D1-D5-P system compactified on T^5 in the five-dimensional case, and to a D1-D5-Kaluza-Klein monopole (KKM) system in the four-dimensional case. Although smooth geometries have been constructed only for very special non-supersymmetric states, this has already led to interesting physics; these geometries are unstable [14], and this instability can be precisely reproduced by studying the dynamics of the corresponding quantum state of the brane system [15].

A next step in the construction of smooth non-supersymmetric geometries would be to obtain smooth solutions which are asymptotically flat in four dimensions corresponding to a four-charge D1-D5-P-KKM system. This extension was not undertaken in [13] because the additional charge seemed to lead to daunting additional complexities. The aim of this paper is to show that the solution with D1-D5-P-KKM charges is in fact related to the D1-D5-KKM solution obtained in [13] by a coordinate transformation, using the analogue of the transformation studied in [16] in the supersymmetric case.

This coordinate transformation is related to spectral flow, which has played an important part in the understanding of these smooth geometries from the outset. When the first simple examples of smooth geometries were constructed in [4], they were related to states obtained by spectral flow from the Neveu-Schwarz ground state in the dual CFT, and this spectral flow was identified with a coordinate transformation of the near-core AdS_3 region in the spacetime geometry. This coordinate transformation was then exploited in the construction of further examples of solutions which were asymptotically flat in five dimensions [17]. The near-core regions of these solutions were related by a coordinate transformation, but the full asymptotically flat solutions were physically distinct. The remarkable realisation of [16] is that once we compactify a further direction on a circle by adding a Kaluza-Klein monopole charge to obtain solutions which are asymptotically flat in four dimensions, the spectral flow transformation which preserves supersymmetry is realised as a coordinate transformation for the full asymptotically flat solution.

The solutions considered in [16] can be written in terms of six-dimensional metrics obtained by a trivial Kaluza-Klein reduction from ten dimensions on a T^4, and describe a geometry sourced by a string-like extended object in the six-dimensional theory. The solutions that are considered are supersymmetric solutions which are asymptotically flat in four dimensions. There are therefore two directions in the six-dimensional solution which have finite size at large distances. In [16], it was shown that a coordinate transformation which mixes up these two Kaluza-Klein circles can be interpreted as spectral flow from the point of view of the dual CFT description of the string sourcing the geometry. From the point of view of the CFT description, this coordinate transformation mixes charges which would be interpreted as R-charges with the Virasoro generators $L_0, \bar{L}_0$, so it is naturally interpreted as a spectral flow automorphism of the CFT.[1]

A particular example considered in [16] is to use this spectral flow to map a two-charge supertube to a bubbling three-charge geometry. Since the general two-charge supertube

[1] Although the transformation is purely a coordinate transformation in the six-dimensional description, it will modify the asymptotic moduli of the solution. Thus, with boundary conditions that fix the asymptotic metric, this coordinate transformation is a global symmetry of the theory, rather than a gauge symmetry, and we can think of the solutions it relates as physically distinct.

solution has an arbitrary profile for the tube, this can be used to construct new infinite families of three-charge solutions. Our interest in the non-supersymmetric case is in the analogue of the simplest case, when we consider a round supertube and the corresponding three-charge solution.

In this paper, we will consider the analogue of the coordinate transformation used in [16] for the non-supersymmetric geometries considered in [13]. For non-supersymmetric geometries, we can consider acting with spectral flow independently on the left and on the right; these transformations are related to two independent coordinate transformations in the near-core AdS_3 region. However, we find that as in the supersymmetric case, only a single combination extends to a coordinate transformation of the full asymptotically flat solution. The coordinate transformation is labelled by a single integer parameter. This coordinate transformation adds an additional momentum charge to the solutions, and we show that it reproduces precisely the expected D1-D5-P-KKM solutions, corresponding to the three-charge D1-D5-P solutions of [11] sitting at the core of a Kaluza-Klein monopole.

In section 2, we give a brief review of the non-supersymmetric geometries of [13]. Section 3 contains the main result of our paper, showing that a spectral flow coordinate transformation can be used to obtain the expected D1-D5-P-KKM solution.

2 The non-supersymmetric microstates of the D1-D5-KK system

We will now briefly review the structure of the smooth non-supersymmetric solutions carrying D1- D5- and Kaluza-Klein (KK) monopole charges constructed in [13], which we want to apply this transformation to. These solutions are asymptotically flat in four dimensions, and there is a family of smooth solutions labelled by a single integer parameter. The metric for the smooth solutions is

$$\begin{aligned} ds_{10}^2 = & \left(\tilde{H}_1\tilde{H}_5\right)^{-1/2}\left[A\left(dy+s_1s_5\mathcal{B}\right)^2 - G\left(dt+c_1c_5\mathcal{A}\right)^2\right] \\ & + \left(\tilde{H}_1\tilde{H}_5\right)^{1/2}\left[\frac{f^2}{AG}\left(dz+\omega^1\right)^2+\frac{d\rho^2}{\Delta}+d\theta^2+\frac{\Delta}{f^2}\sin^2\theta d\phi^2\right] \\ & + \left(\frac{\tilde{H}_1}{\tilde{H}_5}\right)ds_{T^4}^2 \end{aligned} \tag{2.1}$$

where

$$\mathcal{A} = \omega^0 - \frac{C}{G}\left(dz+\omega^1\right), \tag{2.2}$$

$$\mathcal{B} = -V_0\left(dz+\omega^1\right)+\kappa_0^1, \tag{2.3}$$

$$\tilde{H}_{1,5} = A+(A-G)\,s_{1,5}^2, \tag{2.4}$$

$$G = A\left(1-H\right) = \frac{Af^2-C^2}{B}. \tag{2.5}$$

The metric functions are

$$
\begin{aligned}
\Delta &= \rho^2 - \rho_0^2, && (2.6)\\
f^2 &= \left(\rho^2 - \rho_0^2\right) + \rho_0^2 n^2 \sin^2\theta, && (2.7)\\
A &= f^2 + 2p\left[(\rho-\rho_0) + n^2\rho_0\left(1+\cos\theta\right)\right], && (2.8)\\
B &= f^2 + 2\frac{\rho_0\left(p+\rho_0\right)\left(n^2-1\right)}{\left(p-\rho_0\left(n^2-1\right)\right)}\left[(\rho-\rho_0) + n^2\rho_0\left(1-\cos\theta\right)\right], && (2.9)\\
C &= \frac{2\rho_0\sqrt{\rho_0\left(\rho_0+p\right)}}{n}\left(n^2-1\right)\left(p-\rho_0\left(n^2-1\right)\right)\left[(\rho-\rho_0)+(\rho_0+p)\left(1-\cos\theta\right)\right], && (2.10)\\
\omega^0 &= \frac{2J\sin^2\theta\left(\rho-\rho_0\right)}{f^2}d\phi, && (2.11)\\
\omega^1 &= \frac{2}{f^2}\sqrt{p\left(p-\rho_0\left(n^2-1\right)\right)}\left[(\rho^2-\rho_0^2)\cos\theta - \frac{\rho_0 p n^2}{(p-\rho_0(n^2-1))}\left(\rho-\rho_0\right)\sin^2\theta - n^2\rho_0^2\sin^2\theta\right]d\phi, && (2.12)
\end{aligned}
$$

and

$$
\begin{aligned}
V_0 &= -\frac{n\left(n^2-1\right)}{A}\sqrt{\frac{\rho_0^3\left(p+\rho_0\right)}{p\left(p-\rho_0\left(n^2-1\right)\right)^3}}\left[f^2+2p\left(\rho+p+\left(p+\rho_0\right)\cos\theta\right)\right], && (2.13)\\
\kappa_0^1 &= \frac{2n\sqrt{\rho_0(p+\rho_0)}}{(p-\rho_0(n^2-1))}\frac{\sin^2\theta}{f^2}\left(\begin{array}{c}\frac{\rho_0\left(n^2-1\right)}{(p-\rho_0(n^2-1))}\left(p^2+2p\rho_0-\rho_0^2\left(n^2-1\right)\right)\left(\rho-\rho_0\right)\\ +2\rho_0^2(n^2-1)\left(\rho_0+p\right)\end{array}\right), && (2.14)\\
\kappa_0^0 &= -\frac{2}{f^2}\frac{\rho_0\left(p+\rho_0\right)\left(n^2-1\right)}{\left(p-\rho_0\left(n^2-1\right)\right)}\left[\left(\rho^2-\rho_0^2\right)\cos\theta + \left(p\rho-\rho_0^2\left(n^2-1\right)\right)\sin^2\theta\right], && (2.15)
\end{aligned}
$$

where

$$
J^2 = \frac{\rho_0^3 p\left(\rho_0+p\right)n^2\left(n^2-1\right)^2}{\left(p-\rho_0\left(n^2-1\right)\right)}. \tag{2.16}
$$

The determinant of the metric is

$$
g = -\frac{\tilde{H}_1^3}{\tilde{H}_5}\sin^2\theta. \tag{2.17}
$$

It is convenient to introduce the combinations

$$
P^2 = \frac{p\left(p^2+m^2\right)}{(p+q)}, \quad Q^2 = \frac{q\left(q^2+m^2\right)}{(q+p)}, \tag{2.18}
$$

where

$$
q = \frac{\rho_0(p+\rho_0)(n^2-1)}{(p-\rho_0(n^2-1))}. \tag{2.19}
$$

The charges of the four-dimensional asymptotically flat solution are then

$$
\begin{aligned}
\mathcal{M} &= \frac{1}{2}\left[p+q\left(1+s_1^2+s_5^2\right)\right]\\
\mathcal{P} &= P, \quad \mathcal{Q} = Qc_1c_5,\\
\mathcal{J} &= Jc_1c_5, \quad \mathcal{Q}_i = qs_ic_i, \quad i=1,5,
\end{aligned} \tag{2.20}
$$

where $\mathcal{P}, \mathcal{Q}, \mathcal{Q}_1$ and $\mathcal{Q}_5$ are the KK monopole, KK electric, D1 and D5 charges.

The metric has coordinate singularities at $\rho = \rho_0$ and $\theta = 0, \pi$. The determinant of the metric at constant ρ and t is

$$g_{(\rho t)} = \frac{\tilde{H}_1^2}{\tilde{H}_5^2 f^2}(\rho - \rho_0) \sin^2\theta \left[A(\rho+\rho_0)\left(Bc_1^2c_5^2 - f^2(c_1^2s_5^2 + s_1^2c_5^2) + \frac{Gf^2}{A}s_1^2s_5^2\right) - 2Jc_1^2c_5^5\right], \tag{2.21}$$

so at these singularities, a spatial isometry direction is degenerating. To make these singularities smooth origins, we need to make appropriate identifications so that the direction which is degenerating is compact with an appropriate period. This imposes the identifications

$$(y, z, \phi) \sim (y, z - 4\pi\mathcal{P}, \phi + 2\pi) \sim (y, z + 4\pi\mathcal{P}, \phi + 2\pi) \sim (y - 2\pi n R_y, z + 4\pi n\mathcal{P}, \phi + 2\pi n), \tag{2.22}$$

where

$$R_y = 4q\frac{\sqrt{q}\sqrt{p+q}}{\sqrt{q^2+m^2}}s_1s_5. \tag{2.23}$$

The first two identifications in (2.22) guarantee smoothness at $\theta = 0, \pi$, and the last guarantees smoothness at $\rho = \rho_0$.

To facilitate the comparison to the supersymmetric case, we introduce "light-cone coordinates" u, v defined by

$$t = \frac{1}{\sqrt{2}}(u+v), \quad y = \frac{1}{\sqrt{2}}(u-v), \tag{2.24}$$

In these coordinates,

$$\begin{aligned} ds_{10}^2 = &\left(\tilde{H}_1\tilde{H}_5\right)^{-1/2}\left(\frac{A-G}{2}\right)\left[(du+\beta)^2 + (dv+\omega)^2\right] \\ &- \left(\tilde{H}_1\tilde{H}_5\right)^{-1/2}(A+G)(dv+\omega)(du+\beta) \\ &+ \left(\tilde{H}_1\tilde{H}_5\right)^{1/2}\left[\frac{f^2}{AG}\left(dz+\omega^1\right)^2 + \frac{d\rho^2}{\Delta} + d\theta^2 + \frac{\Delta}{f^2}\sin^2\theta d\phi^2\right] \\ &+ \left(\frac{\tilde{H}_1}{\tilde{H}_5}\right)ds_{T^4}^2, \end{aligned} \tag{2.25}$$

where we define

$$\zeta_1 = s_1s_5\mathcal{B} = \frac{1}{\sqrt{2}}(\beta - \omega), \quad \zeta_2 = c_1c_5\mathcal{A} = \frac{1}{\sqrt{2}}(\beta + \omega), \tag{2.26}$$

and β and ω are given by

$$\beta = \frac{1}{\sqrt{2}}(\zeta_1 + \zeta_2) = -\eta_1\left(dz + \omega^1\right) + \eta_2, \tag{2.27}$$

$$\omega = \frac{1}{\sqrt{2}}(\zeta_2 - \zeta_1) = \eta_3\left(dz + \omega^1\right) + \eta_4, \tag{2.28}$$

$$\tag{2.29}$$

where

$$\eta_1 = \frac{1}{\sqrt{2}} \left(s_1 s_5 V_0 + c_1 c_5 \frac{C}{G} \right), \tag{2.30}$$

$$\eta_2 = \frac{1}{\sqrt{2}} \left(c_1 c_5 \omega^0 + s_1 s_5 \kappa_0^1 \right), \tag{2.31}$$

$$\eta_3 = \frac{1}{\sqrt{2}} \left(s_1 s_5 V_0 - c_1 c_5 \frac{C}{G} \right), \tag{2.32}$$

$$\eta_4 = \frac{1}{\sqrt{2}} \left(c_1 c_5 \omega^0 - s_1 s_5 \kappa_0^1 \right). \tag{2.33}$$

2.1 BPS case

The solution is supersymmetric for $n = 1$, where $m = 0$ and we must take the $\delta_i \to \infty$ to hold the charges $\mathcal{Q}_i$ fixed. This case therefore requires a slightly separate discussion. When $n = 1$, $C = 0$ and $B = f^2 = \rho^2 - \rho_0^2 \cos^2\theta$, so $G = A$, but

$$\left(1 - \frac{G}{A}\right) \sinh^2 \delta_i = \frac{2\mathcal{Q}_i}{\rho + \rho_0 \cos\theta}, \tag{2.34}$$

so $\tilde{H}_i = A\left(1 + \frac{2\mathcal{Q}_i}{\rho + \rho_0 \cos\theta}\right)$. Similarly, the one-forms ω and β in (2.25) will have finite limits. We also have

$$\frac{A}{f^2} = 1 + \frac{2p}{\rho - \rho_0 \cos\theta}. \tag{2.35}$$

It is then useful to introduce the new coordinates

$$\tilde{r} = \rho - \rho_0 \cos\theta, \quad \cos\tilde{\theta} = \frac{\rho\cos\theta - \rho_0}{\rho - \rho_0 \cos\theta} \tag{2.36}$$

and define the parameters

$$c = 2b, \quad Q_K = 2\mathcal{P}, \quad Q_{Ke} = 2\mathcal{Q}, \quad Q_i = 2\mathcal{Q}_i, \quad i = 1, 5. \tag{2.37}$$

We then have

$$V = \frac{A}{f^2} = 1 + \frac{Q_K}{\tilde{r}}, \quad Z_i = \frac{\tilde{H}_i}{A} = 1 + \frac{Q_i}{\tilde{r}_c}, \quad i = 1, 5 \tag{2.38}$$

where

$$\tilde{r}_c = \sqrt{\tilde{r}_c^2 + c^2 + 2c\tilde{r}\cos\tilde{\theta}}. \tag{2.39}$$

The metric (2.25) in the supersymmetric case $n = 1$ then takes the form

$$\begin{aligned} ds_{10}^2 = & -\frac{2}{H}\,(dv + \omega)\,(du + \beta) + HV^{-1}\left(dz + \omega^1\right) \\ & + HV\left(d\tilde{r}^2 + \tilde{r}^2 d\tilde{\theta}^2 + \tilde{r}^2 \sin^2\tilde{\theta}^2 d\phi^2\right) + \left(\frac{Z_1}{Z_5}\right) ds_{T^4}^2, \end{aligned} \tag{2.40}$$

where $H = \sqrt{Z_1 Z_5}$, reproducing the form used for example in [16]. Note however that the coordinates here are not exactly the same as in [16]; in particular, by (2.22), in (2.40) the z coordinate has period $4\pi Q_K$, and the v coordinate has period $2\pi n R_y$.

2.2 Near-core limit

The solution reviewed above is supposed to be interpreted as the familiar smooth D1-D5 solution of [11] which is asymptotically flat in five dimensions, sitting at the core of a Kaluza-Klein monopole which converts it into a solution which is asymptotically flat in four dimensions. In [13], this was argued by showing that the metric (2.1) has a near-core limit where it reduces to an $AdS_3 \times S^3$ geometry of the same form as is obtained in the near-core limit of the five-dimensional solution of [11]. We will now briefly review this near-core limit.

The appropriate limit is to take $\rho_0 \to 0$ holding p and the D1, D5 brane charges $\mathcal{Q}_i$ fixed. This limit will scale $\mathcal{Q}$ and $\mathcal{J}$ to zero, so it is distinct from the supersymmetric limit. As we take this limit, we scale the coordinates so as to zoom in on a core region in the geometry, scaling ρ like ρ_0, and the identification on the y coordinate scales like $1/\sqrt{\rho_0}$. We therefore define new coordinates by

$$\rho = \rho_0(1+2R^2), \quad y = \frac{\ell^2 \varphi}{4\sqrt{p\rho_0}}, \quad t = \frac{\tau}{4\sqrt{p\rho_0}}, \quad z = p\psi, \tag{2.41}$$

and take the limit keeping R, φ, τ finite. In this limit the metric (2.1) becomes $AdS_3 \times S^3$ geometry at least locally. To write the sphere in standard coordinates, we additionally define the new coordinates

$$\theta = 2\bar{\theta}, \quad \bar{\psi} = \frac{1}{4}(2\phi+\psi), \quad \bar{\phi} = \frac{1}{4}(2\phi-\psi). \tag{2.42}$$

The metric is then

$$\begin{aligned} ds^2 \approx & -\frac{R^2+1}{\ell^2}d\tau^2 + \frac{\ell^2 dR^2}{R^2+1} + \ell^2 R^2 d\varphi^2 \\ & + \ell^2\left(d\bar{\theta}^2 + \cos^2\bar{\theta}(d\bar{\psi}+n d\varphi)^2 + \sin^2\bar{\theta}\left(d\bar{\phi} - \frac{n}{\ell^2}d\tau\right)^2\right) + \sqrt{\frac{\mathcal{Q}_1}{\mathcal{Q}_5}}ds^2_{T^4}. \end{aligned} \tag{2.43}$$

In the near-core limit,

$$nR_y = 4\sqrt{\frac{p}{\rho_0}}\sqrt{\mathcal{Q}_1\mathcal{Q}_5}, \tag{2.44}$$

so the identifications (2.22) become in these coordinates simply $\bar{\psi} \sim \bar{\psi} + 2\pi$, $\bar{\phi} \sim \bar{\phi} + 2\pi$, and $(\varphi, \bar{\psi}) \sim (\varphi - 2\pi, \bar{\psi} + 2\pi n)$. If these are the fundamental identifications, the spacetime is then globally $AdS_3 \times S^3$.

3 Spectral flow

Let us now consider the construction of new solutions by acting on the non-supersymmetric geometry (2.1) with a spectral flow coordinate transformation. As in [16], we consider the coordinate transformation

$$z \to z + \gamma v, \tag{3.1}$$

where γ is a parameter, and we define the spectral flow using the coordinates of (2.25). It is then clear that the spectral flow we consider here will coincide with the one studied in [16] in

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the supersymmetric case $n = 1$, where it corresponds to the simple example we mentioned in the introduction, relating the round two-charge supertube in the Kaluza-Klein monopole background to a three-charge bubbling solution in the same monopole background.

In general, acting on the non-supersymmetric metric (2.25) with the spectral flow (3.1), we will obtain a new solution

$$ds_6^2 = -\frac{2}{\tilde{H}}(dv + \tilde{\omega})\left[du + \tilde{\beta} + \frac{\tilde{F}}{2}(dv + \tilde{\omega})\right] + \tilde{H}\tilde{V}^{-1}\left(dz + \tilde{\omega}^1\right) \\ + \tilde{H}\tilde{V}\left[d\tilde{r}^2 + \tilde{r}^2 d\tilde{\theta}^2 + \tilde{r}^2 \sin^2\tilde{\theta} d\phi^2\right], \tag{3.2}$$

where

$$\tilde{\omega} = (1+\gamma\eta_3)^{-1}\omega, \qquad \tilde{H} = (1+\gamma\eta_3)^{-1} H, \qquad \tilde{V} = (1+\gamma\eta_3) V, \tag{3.3}$$

$$\tilde{\omega}^1 = \omega^1 - \gamma\eta_4, \qquad \tilde{F} = -2\gamma\eta_1 - \frac{\gamma^2 H^2 V^{-1}}{(1+\gamma\eta_3)}, \tag{3.4}$$

$$\tilde{\beta} = \beta + \frac{\gamma\eta_1}{(1+\gamma\eta_3)}\omega + \frac{\gamma^2 H^2 V^{-1}}{(1+\gamma\eta_3)^2}\omega - \frac{\gamma H^2 V^{-1}}{(1+\gamma\eta_3)}\left(dz + \omega^1\right). \tag{3.5}$$

As in the supersymmetric case, this new solution has an additional charge given by the Kaluza-Klein gauge potential $\tilde{F}$, corresponding to a momentum (Kaluza-Klein electric) charge along the circle that the D1 and D5 branes wrap. Note that this is not the same as the Kaluza-Klein electric charge which is already present in the solution (2.25), which is associated with the Kaluza-Klein gauge field coming from the z circle. We therefore refer to the resulting solution as a four-charge D1-D5-P-KKM solution.

In general, this solution will not have the same asymptotics as the solution that we started with. As in the supersymmetric case, we need to quantise γ to ensure that the spectral flow transformation preserves the identifications (2.22). Starting from the identifications (2.22) in the original coordinates and applying the transformation $z \to z + \gamma v$, the identifications in the new coordinates are

$$(y,z,\phi) \sim (y, z-4\pi\mathcal{P}, \phi+2\pi) \sim (y, z+4\pi\mathcal{P}, \phi+2\pi) \sim (y-2\pi n R_y, z+4\pi n\mathcal{P}-2\pi n R_y\gamma, \phi+2\pi n). \tag{3.6}$$

we want these to be the same as the identifications (2.22) for the new coordinates. This will be true if $z \sim z - 2\pi n R_y \gamma$. Since (3.6) implies $z \sim z + 8\pi\mathcal{P}$, this requires

$$\gamma = \frac{4m\mathcal{P}}{nR_y} \tag{3.7}$$

for some integer m. This is the analogue for our case of the restriction of γ to even integers in the supersymmetric case, and taking into account the difference in our normalization of z, v, it will reduce to that identification in the supersymmetric case.

Spectral flow then gives us a solution labelled by the D1 and D5 brane and Kaluza-Klein monopole charges, the size of the y circle R_y, and two integer parameters m, n.[2]

[2] We can think equivalently think of these solutions as labelled by the D1, D5, P charges, the Kaluza-Klein monopole charge and the two integer parameters m, n.

This is exactly the number of solutions we would expect if we were to put the three-charge D1-D5-P smooth solutions of [11] at the core of a Kaluza-Klein monopole. We will relate the solution here to the solution of [11] by considering the near-core limit, as was done for the solution without the momentum charge in [13].

Since the near-core limit has already been worked out in the original coordinates before we perform the spectral flow in [13], as reviewed in section 2.2, all we need to do is to consider the action of the spectral flow transformation in this near-core region. Using the coordinate transformations (2.41), (2.42), the spectral flow transformation (3.1) becomes, in this near-core region,

$$\bar{\psi} \to \bar{\psi} + \frac{\gamma}{16p\sqrt{p\rho_0}} \left(\tau - \ell^2 \varphi\right), \quad \bar{\phi} \to \bar{\phi} - \frac{\gamma}{16p\sqrt{p\rho_0}} \left(\tau - \ell^2 \varphi\right). \tag{3.8}$$

In the near-core region, R_y is given by (2.44), so the quantization condition (3.7) becomes

$$\gamma = m\sqrt{p\rho_0} \frac{1}{\sqrt{\mathcal{Q}_1 \mathcal{Q}_5}}, \tag{3.9}$$

and $\ell^2 = 16p\sqrt{\mathcal{Q}_1\mathcal{Q}_5}$, so the spectral flow transformation in the near-core region is

$$\bar{\psi} \to \bar{\psi} + \frac{m}{\ell^2} \left(\tau - \ell^2 \varphi\right), \quad \bar{\phi} \to \bar{\phi} - \frac{m}{\ell^2} \left(\tau - \ell^2 \varphi\right). \tag{3.10}$$

This shows that the near-core limit of the spectral flow coordinate transformation (3.1) agrees with the usual notion of spectral flow for $\mathrm{AdS}_3 \times S^3$ spacetimes, and the near-core metric of the new solution obtained by acting with this spectral flow transformation will be

$$\begin{aligned} ds^2 \approx & -\frac{(1+R^2)}{\ell^2} d\tau^2 + \ell^2 R^2 d\varphi^2 + \frac{\ell^2 dR^2}{R^2+1} + \ell^2 d\bar{\theta}^2 \\ & + \ell^2 \cos^2\bar{\theta} \left(d\bar{\psi} + \frac{m}{\ell^2} d\tau - (m-n)\, d\varphi\right)^2 + \ell^2 \sin^2\bar{\theta} \left(d\bar{\phi} - \frac{1}{\ell^2}(m+n)\, d\tau + m d\varphi\right)^2 \\ & + \sqrt{\frac{\mathcal{Q}_1}{\mathcal{Q}_5}} ds^2_{T^4}. \end{aligned} \tag{3.11}$$

This agrees with the near-core region of the three-charge solution in [11], up to a relabelling of the integer parameters and a trivial shift of $\bar{\psi}, \bar{\phi}$ by terms proportional to τ. Thus, these four-charge D1-D5-P-KKM solutions obtained by (3.1) can indeed be identified with the three-charge solution of [11] sitting at the core of a Kaluza-Klein monopole.

Thus, we have shown that the spectral flow coordinate transformation (3.1) can be used to construct the remaining simple example of a non-supersymmetric solution, the D1-D5-P-KKM solution. We could also consider orbifolds of the solution as in [11].

4 Conclusions

We have shown that spectral flow can be realised as a coordinate transformation for non-supersymmetric solutions which are asymptotically flat in four dimensions, generalising the observation of [16] in the supersymmetric case. We have focused on the spectral flow of the known smooth solutions, obtained in [13], thus completing the catalogue of "simple" smooth solutions, corresponding to geometries with two centers in the four-dimensional metric. The explicit construction of these metrics will be useful for future studies of the map between these smooth solutions and CFT microstates. Unlike in the supersymmetric case, when we consider solutions which involve an orbifold of the S^3 in the near-horizon region, the map onto CFT states is not understood even for these simple two-center solutions [11].

The same arguments could however be applied to any new non-supersymmetric solutions that are constructed. This simplifies the construction of further non-supersymmetric solutions by removing the requirement to consider the additional Kaluza-Klein momentum charge which can be added by spectral flow. This is an important simplification; the extension of the direct analysis of [13] to the case with Kaluza-Klein momentum charge is extremely laborious. However, the problem of constructing further non-supersymmetric solutions is extremely challenging, even in the simpler setting with just D1, D5 and KK monopole charges, and this remains one of the most important open problems for this area. Further progress on these solutions will require the development of new techniques.

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